STUDY OF THE SIMPLEST STATISTICAL CHARACTERISTICS OF A TURBULENT FLOW TEMPERATURE DISTRIBUTION

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UDC 532.517.4

The degeneracy of the temperature distribution in homogeneous nonisotropic turbulence is examined. The relations obtained are verified experimentally.

The mean square of the temperature fluctuations in homogeneous turbulence is described by the well-known balance equation

$$\frac{d\overline{t^2}}{d\tau} + 12\varkappa D_{t0} = 0, \tag{1}$$

where $D_{t0} = -{}^1/_6\,(\Delta R_t)_0$ is a function characterizing the leveling of the temperature fluctuations. The equation for D_{t0} is easily obtained from the equation for the two-point correlation of temperature fluctuations (cf., e.g., Corrsin [1]). This equation can be written in the form

$$\frac{d}{d\tau} D_{t0} + \frac{5}{\sqrt{3}}\left(S_t + \frac{2}{3} S_\varkappa\right) D_{t0} D_{u0}^{1/2} = 0, \tag{2}$$

where $D_{u0}^{1/2} = -{}^1/_5\,(\Delta R_u)_0$ is the vorticity of the field of velocity fluctuations. In this equation the coefficients S_t and $S_\varkappa$ are dimensionless quantities composed of derivatives with respect to $\vec{\xi}$ of two-point temperature and velocity correlations:

$$S_t = \frac{1}{5\sqrt{3}} \frac{\left(\frac{\partial}{\partial \xi_i} \Delta_\xi \overline{u_i t t'}\right)_0}{D_{t0}(D_{u0})^{1/2}}; \tag{3}$$

$$S_\varkappa = \frac{3\sqrt{3}}{5} \varkappa \frac{(\Delta_\xi D_t)_0}{D_{t0} D_{u0}^{1/2}}. \tag{4}$$

Physically, the first term in Eq. (2) represents the total time rate of change of D_{t0}, the second represents diffusion, and the third, the leveling of D_{t0} (dissipation) as the result of the thermal diffusivity of the medium.

We note that for homogeneous isotropic turbulence Eqs. (1) and (2) go over into the equations of an isotropic temperature field (the second of these can be obtained from Corrsin's equation [1])

$$\frac{d\overline{t^2}}{d\tau} + 12\varkappa \frac{\overline{t^2}}{\lambda_t^2} = 0, \tag{1*}$$

$$\frac{d}{d\tau}\left(\frac{\overline{t^2}}{\lambda_t^2}\right) + \frac{5}{\sqrt{3}}\left(S_t^* + \frac{2}{3} S_\varkappa^*\right) \frac{\overline{t^2}}{\lambda_t^2} \cdot \frac{q}{\lambda_u} = 0, \tag{2*}$$

where $\lambda_t^2 = \frac{\overline{t^2}}{D_{t0}^*}$, $\lambda_u^2 = \frac{\overline{q^2}}{D_{u0}^*}$ are, respectively, the squares of the scales of the degeneracy of the turbulent fluctuations of the temperature and velocity. The coefficients (3) and (4) have the form

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Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 30, No. 6, pp. 1025-1034, June, 1976. Original article submitted February 24, 1975.

$$S_t^* = \frac{\overline{\left(\frac{\partial t}{\partial x_1}\right)^2 \cdot \frac{\partial u_1}{\partial x_1}}}{\overline{\left(\frac{\partial t}{\partial x_1}\right)^2} \cdot \overline{\left(\frac{\partial u_1}{\partial x_1}\right)^2}^{1/2}},$$

$$S_\varkappa^* = \varkappa \frac{\overline{\left(\frac{\partial^2 t}{\partial x_1^2}\right)^2}}{\overline{\left(\frac{\partial t}{\partial x_1}\right)^2} \cdot \overline{\left(\frac{\partial u_1}{\partial x_1}\right)^2}^{1/2}}, \qquad (5)$$

where u_1 is the x_1 component of the velocity fluctuation.

The validity of Eqs. (1) and (2) as a description of the temperature distribution for homogeneous turbulence is determined in the last analysis by the values of the coefficients S_t and $S_\varkappa$ as functions of the parameters which determine the temperature distribution for homogeneous turbulence, i.e., on the local Reynolds and Péclet numbers:

$$\mathrm{Re} = \frac{q l_u}{\nu}, \quad \mathrm{Pl} = \frac{q l_t}{\varkappa},$$

where l_u and l_t are certain length scales characterizing the velocity and temperature fields.

As far as we know there are no published reports of direct measurements of S_t and $S_\varkappa$; we know only of the paper by Yaglom [2] where S_t^* is estimated on the basis of the Betchov inequality.

We estimate the values of S_t and $S_\varkappa$ on the basis of the known laws of degeneracy of a uniform isotropic temperature field.

For the final stage of degeneracy of the temperature field ($\mathrm{Pl} \ll 1$) the diffusion term in Eq. (2*) can be neglected. Thus we have

$$\frac{d}{d\tau}\overline{t^2} + 12\varkappa \frac{\overline{t^2}}{\lambda_t^2} = 0,$$

$$\frac{d}{d\tau}\lambda_t^2 + \left(12 - \frac{10}{3\sqrt{3}} S_\varkappa^* \frac{P_\lambda^2}{R_\lambda} \cdot \frac{1}{\sigma}\right)\varkappa = 0.$$

If we assume that

$$S_\varkappa^* \frac{P_\lambda^2}{R_\lambda} \cdot \frac{1}{\sigma} = S_{TDIS}^* = \text{const},$$

the system of equations has the solution

$$\overline{t^2}(\lambda_t^2)^{1/\beta} = \text{const}, \text{ where } \beta = \frac{5}{18\sqrt{3}} S_t^* - 1.$$

Since Corrsin's invariant [1] is valid in the final stage of degeneracy of a uniform isotropic temperature field

$$\overline{t^2}\lambda_t^3 = \text{const},$$

we must set $\beta = \frac{2}{3}$, which gives

$$S_{TDIS}^* = 6\sqrt{3}. \qquad (6)$$

Thus for very small values of the Péclet number the coefficient S_{TDIS}^* is a universal constant, and, consequently, $S_\varkappa^*$ is a function of the Reynolds, Péclet, and Prandtl numbers†:

$$S_\varkappa^* = 6\sqrt{3}\,\sigma \frac{R_\lambda}{P_\lambda^2}. \qquad (7)$$

We now turn to the limiting case of very large values of the local Péclet number. In this case the diffusion transport and "dissipation" in Eq. (2*) are of the same order of magnitude. Introducing the length scale of the dissipation of the temperature field

† We derived a similar result earlier [3] for the corresponding coefficient S_{UDIS}^* of the velocity field.

$$L_t = \frac{q\overline{t^2}}{\varkappa D_{t0}^*},$$

we write Eqs. (1) and (2) in the form

$$\frac{d}{d\tau}\overline{t^2} + 12g\frac{\overline{t^2}}{L_t} = 0,$$

$$\frac{d}{d\tau}L_t + 12g\left\{1 - \frac{5}{12}\left[\frac{1}{\sqrt{3}}\left(S_t^* + \frac{2}{3}S_\varkappa^*\right) - \frac{1}{R_L^{1/2}}\right]\frac{P_L}{R_L^{1/2}}\,\frac{1}{\sigma}\right\} = 0.$$

If we assume the validity of the relation

$$\left(S_t^* + \frac{2}{3}S_\varkappa^*\right)\frac{P_L}{R_L^{1/2}}\cdot\frac{1}{\sigma} - \sqrt{3}\,\frac{P_L}{R_L}\cdot\frac{1}{\sigma} = \frac{2}{3}\,S_{TDIS}^* - S_{TDIF}^* - \sqrt{3}\,\frac{P_L}{R_L}\cdot\frac{1}{\sigma} = \text{const},$$

where

$$S_{TDIS}^* = S_\varkappa^*\frac{P_L}{R_L^{1/2}}\cdot\frac{1}{\sigma};\quad S_{TDIF}^* = -S_t^*\,\frac{1}{\sigma}\cdot\frac{P_L}{R_L^{1/2}},$$

the system of equations has the following solution:

$$\overline{t^2}\,L_t^{1/\beta} = \text{const},$$

where

$$\beta = \frac{5}{12\sqrt{3}}\left(\frac{2}{3}S_{TDIS}^* - S_{TDIF}^* - \frac{\sqrt{3}}{\sigma}\cdot\frac{P_L}{R_L}\right) - 1.$$

Thus the well-known Corrsin solution [1] has been obtained, from which we find $\beta = 1/3$. This means that the following relation holds for large Péclet numbers:

$$\frac{2}{3}\,S_{TDIS}^* - S_{TDIF}^* - \sqrt{3}\,\frac{1}{\sigma}\cdot\frac{P_L}{R_L} = F_2 = \frac{16\sqrt{3}}{5}.$$

Hence there follows a relation between S_t^* and $S_\varkappa^*$

$$\frac{2}{3}S_\varkappa^* + S_t^* = \frac{16\sqrt{3}}{5}\cdot\frac{R_L^{1/2}}{P_L}\,\sigma + \sqrt{3}\,\frac{1}{R_L^{1/2}}. \tag{8}$$

Thus it has been shown that the coefficients in Eq. (2*) are not universal constants either for $\mathrm{Pl} \ll 1$ or $\mathrm{Pl} \gg 1$.

However, the combinations (6) and (8) of these coefficients and the Reynolds and Péclet numbers are universal constants in these limiting cases. The results obtained are valid for a uniform isotropic temperature field. Unfortunately, such an analysis is not possible for a uniform nonisotropic field, since the laws of degeneracy of uniform nonisotropic fields are not known. Therefore, we determine the values of S_{TDIS} and S_{TDIF} experimentally for homogeneous nonisotropic turbulence.

We write the expressions for S_{TDIS} and S_{TDIF} explicitly in terms of the correlation functions

$$S_{TDIS} = \frac{3\sqrt{3}}{5}\cdot\frac{\overline{t^2}(\Delta_\xi D_t)_0}{D_{t0}^2}, \tag{9}$$

$$S_{TDIF} = \frac{1}{5\sqrt{3}}\cdot\frac{\overline{t^2}}{\varkappa}\cdot\frac{\left(-\frac{\partial}{\partial\xi_i}\,\Delta_\xi\,\overline{u_i t t'}\right)_0}{D_{t0}^2}. \tag{10}$$

Using this notation, Eqs. (1) and (2) take the form

$$\frac{d}{d\tau}\,\overline{t^2} + 12\,\frac{\overline{t^2}q}{L_t} = 0, \tag{11}$$

$$\frac{d}{d\tau}\left(\frac{\overline{t^2}q}{L_t}\right) + \frac{5}{\sqrt{3}}\left(\frac{2}{3}S_{TDIS} - S_{TDIF}\right)\frac{\overline{t^2}q^2}{L_t^2} = 0. \tag{12}$$

It is desirable to compare the asymptotic expressions (8) for S_{TDIS} and S_{TDIF} with the experimental values found in a uniform nonisotropic temperature field over a wide range of Reynolds and Péclet numbers.

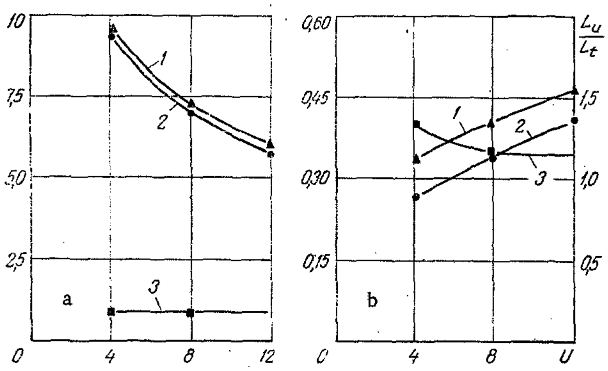

Fig. 1. Scales of temperature and velocity fields as functions of flow velocity: a) microscales in mm: 1) λ_t; 2) λ_u; 3) λ_t/λ_u; b) macroscales in m: 1) Δ_u; 2) Δ_t; 3) L_u/L_t. U, m/sec.

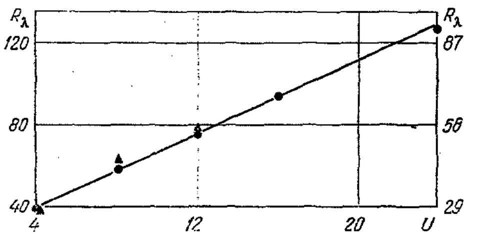

Fig. 2. Reynolds and Péclet numbers as functions of flow velocity.

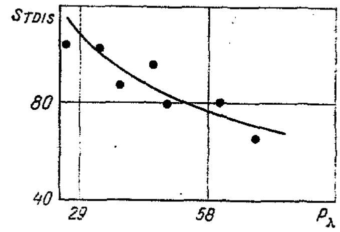

Fig. 3. Dissipative coefficient of temperature field as a function of Péclet number.

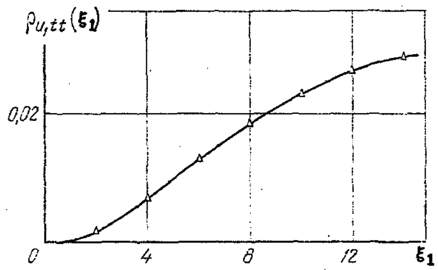

Fig. 4

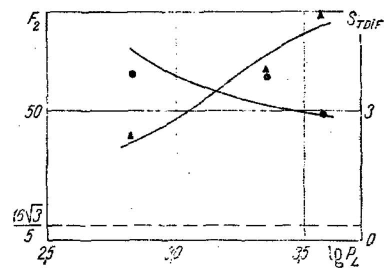

Fig. 5

Fig. 4. Mixed two-point third-order correlation coefficient of velocity and temperature for U = 8 m/sec.

Fig. 5. Diffusion coefficient of temperature field and F_2 as functions of Péclet number.

The experiment was performed for the simplest form of homogeneous nonisotropic turbulence – axisymmetric turbulence. The results of this experiment for the characteristics of the velocity field are given in [3]. A uniform temperature distribution was obtained by using a screen with a 40 × 40 mm square mesh made of Constantan wire and heated by an electric current. The mean-square value of the temperature fluctuations in the working region was 0.2–0.3°C. The experiment was performed with standard apparatus made by the DISA company (hot-wire anemometers with high-pass and low-pass filters, linearizers, an effective value voltmeter), resistance thermometers, and apparatus to transform and feed analog signals into an analog computer. The apparatus is described in detail in [4].

It follows from Eqs. (9) and (10) that the experimental determination of S_{TDIS} and S_{TDIF} requires the measurement of two-point temperature and velocity correlation functions in three directions and the calculation of appropriate derivatives up to and including the fourth order. To avoid repeated calibrations of the apparatus the normalized correlation functions, that is, the correlation coefficients, were measured. In measuring the transverse correlation coefficients, transducers were placed at two points in space and one of them was displaced in the ξ_2 and ξ_3 directions (transverse coordinates). The initial distance between transducers was measured with a microscope. Information on velocity and temperature fluctuations was fed into the analog computer automatically for the calculation of the corresponding correlations. The hypothesis of "frozen turbulence" was used to calculate the longitudinal correlation coefficients $\rho_{tt'}(\xi_1, \xi_2)$; i.e., the space–time coefficients $\rho_{tt'}(\tau, \xi_2)$ rather than the spatial coefficients were measured. The step in the longitudinal coordinate was assigned by using a sampling generator $\xi_1 = \frac{U}{f} = U_1\tau$, where f is the frequency of the sampling and τ is the difference in time. For a more accurate measurement of two-point temperature correlations the amplitude–frequency and phase characteristics of the resistance thermometers were flattened.

The measurement of two-point cross-correlation coefficients $\rho_{u_1tt'}(\tau)$ is an interesting procedural problem, since in this case the hot-wire anemometer output signal depends on both the temperature and velocity fluctuations. The output signals of a hot-wire anemometer and a resistance thermometer with their transducers located in the immediate vicinity of one another have the form $l_1 = \alpha u_1 - \beta t + m$, $l_2 = \gamma t + k$, $l_3 = \alpha u_1 + m$, where l_1 is the hot-wire anemometer signal in a nonisothermal flow, l_2 is the resistance thermometer signal, l_3 is the hot-wire anemometer signal in an isothermal flow, and α, β, γ are sensitivity coefficients; m and k are, respectively, the hot-wire anemometer and resistance thermometer noise. If it is assumed that the noise and useful signals are not correlated, the measured correlation coefficient can be written in the form

$$\rho_{meas} = \rho_{u_1tt(\tau)} \frac{\alpha\gamma^2 \sqrt{\overline{u_1^2}} \sqrt{\overline{t^2}}}{\sqrt{\overline{l_2^2}\ \overline{l_2^2}}} - \rho_{t^2t(\tau)} \frac{\gamma^2\beta \left(\sqrt{\overline{t^2}}\right)^3}{\sqrt{\overline{l_1^2}\ \overline{l_2^2}}} =$$

$$= \rho_{u_1tt(\tau)} \frac{\sqrt{\overline{l_3^2} - \overline{m^2}}\ (\overline{l_2^2} - \overline{k^2})}{\sqrt{\overline{l_1^2}\ \overline{l_2^2}}} - \rho_{t^2t(\tau)} \frac{\left(1 - \frac{\overline{k^2}}{\overline{l_2^2}}\right)\beta\sqrt{\overline{t^2}}}{\sqrt{\overline{l_1^2}}},$$

i.e.,

$$\rho_{u_1tt(\tau)} = \frac{\sqrt{\overline{l_1^2}}}{\sqrt{\overline{l_3^2} - \overline{m^2}}\left(1 - \frac{\overline{k^2}}{\overline{l_3^2}}\right)} + \rho_{t^2t(\tau)} \frac{\sqrt{\overline{l_1^2} - \overline{l_3^2}}}{\sqrt{\overline{l_3^2} - \overline{m^2}}}.$$

For a large signal-to-noise ratio the effect of instrument noise can be neglected. Then the preceding relation can be rewritten in the form

$$\rho_{u_1tt(\tau)} = \rho_{meas} \frac{\sqrt{\overline{l_1^2}}}{\sqrt{\overline{l_3^2}}} + \rho_{t^2t(\tau)} \sqrt{\frac{\overline{l_1^2}}{\overline{l_3^2}} - 1}. \tag{13}$$

An extra measurement of $\rho_{t^2t(\tau)}$ is performed to calculate $\rho_{u_1tt(\tau)}$. In doing this only the output signal of the resistance thermometer is processed. The extra measured function $\rho_{t^2t(\tau)}$ corresponds to the function appearing in Eq. (13) if the resistance thermometer and the hot-wire anemometer reproduce the temperature fluctuations spectrum identically. The extent of the reproduction of the fluctuations spectrum was estimated by the agreement of the functions $\rho_{tt(\tau)}$ measured by the hot-wire anemometer and the resistance thermometer. Agreement was achieved by flattening the amplitude–frequency and phase characteristics of the resistance thermometer and the hot-wire anemometer.

The derivatives of the correlation functions were calculated by the numerical differentiation formulas for four equally spaced points. The volume of information over which the averaging was performed was chosen so that the statistical error of the calculation of a derivative was no more than 5%. As a rule, from 10^5 to $2 \cdot 10^5$ samplings were required. The time step was selected according to the width of the signal spectrum.

The experimental results are given in Figs. 1-5. The correlation coefficients were measured at 16 points, which was sufficient for the calculation of derivatives with respect to ξ up to and including the fourth order. Using the values obtained for D_t and the root-mean-square values of the temperature fluctuations, the values of the microscale of the temperature field were calculated:

$$\lambda_t = \frac{\sqrt{\overline{t^2}}}{D_t^{1/2}}.$$

This characteristic is shown in Fig. 1. The figure also shows the curve for the microscale of the velocity field

$$\lambda_u = \frac{q}{D_u^{1/2}}$$

taken from [3]. It should be noted that the ratio of the squares of the microscales of the velocity and temperature fields is practically constant, which follows, in particular, from the exact solutions of von Kármán and Howarth [5] and Corrsin [1] for isotropic fields.

Figure 2 shows the range of variation of microscale Reynolds and Péclet numbers: $R_\lambda = q\lambda_u/\nu$, $P_\lambda = q\lambda_t/\varkappa$.

The curve of Fig. 3 shows that S_{TDIS} is a function of the Péclet number and that the values of this function are very far from the value $S_{TDIS}=6\sqrt{3}$ obtained for $P_\lambda \ll 1$, which clearly is due to the lack of isotropy. The accuracy of the calculation of the statistical coefficient S_{TDIS} is characterized by a mean-square error of 12%. The advantage of expressing the error in this way is that the mean-square error has a quite definite value of the confidence coefficient of 0.68, and twice the mean-square error, a value of 0.95. Keeping in mind the range of Péclet numbers involved in the present experiment, the experimental results are extended only to a verification of Eqs. (8), (9), and (10).

We calculated the macroscale L_t of the temperature field by using the value of the vorticity D_t and the magnitudes of the fluctuations of velocity q^2 and temperature $\bar{t}^2$. Figure 1 shows L_t and the macroscale of the velocity field

$$L_u = \frac{q^3}{\nu D_u}$$

as functions of the flow velocity. The figure shows that the macroscales increase with increasing flow velocity, approaching certain asymptotes, while the ratio of the scales remains practically constant. This result is also known as a consequence of the laws of degeneracy of isotropic turbulence for very large Reynolds and Péclet numbers (cf., e.g., Kolmogorov [6] and Corrsin [1]).

The third-order derivatives of the correlation function $\overline{u_1 tt'}$ must be determined to calculate the coefficient S_{TDIF}. If we consider the complete expression for this operator in components, it becomes clear that a direct measurement of it using the apparatus mentioned above is impracticable. To simplify the operator

$$\left(-\frac{\partial}{\partial \xi_j}\Delta_\xi \overline{u_i tt'}\right)_0$$

we used the approximate expression of a homogeneous correlation tensor of the first rank for closely spaced points given in [7]:

$$\overline{u_i tt'} = \frac{qt^2}{\sqrt{3}}\left(\frac{1}{6}R^{(1)}K_{mn}\xi_m\xi_n + \ldots\right)\xi_i,$$

$$m = 1,\ 2,\ 3;\quad n = 1,\ 2,\ 3;\quad K_{mn} = \frac{\overline{u_m u_n}}{\frac{1}{3}q^2};$$

$$R^{(1)} = \frac{\sqrt{3}}{5}\cdot\frac{1}{q\bar{t}^2}\left(\frac{\partial}{\partial \xi_i}\Delta_\xi \overline{u_i tt'}\right)_{\vec{\xi}=0}.$$

By using this expression the coefficient (10) can be written in the form

$$S_{TDIF} = \frac{1}{3}q\,\frac{\bar{t}^{2^2}}{\varkappa}\cdot\frac{\rho^{(1)}}{D_{t0}^2} = 12\sqrt{3}\,\frac{\sqrt{\overline{u_1^2}}}{\varkappa K_{11}}\cdot\frac{\left(\frac{\partial^3}{\partial \xi_1^3}\rho_{u_1tt'}\right)_{\vec{\xi}=0}}{(-\Delta_\xi \rho_{tt'})^2_{\vec{\xi}=0}}.$$

This coefficient was determined by measuring the two-point correlation coefficient $\rho_{u_1tt'}(\xi_1)$ in addition to the previously determined quantities.

In order to do this two transducers were placed at a working point in $\vec{\xi}$ space with coordinates (0, 0, 0). One of the transducers was part of the resistance thermometer system and was sensitive only to temperature fluctuations. The transducer wires (5μ for the hot-wire anemometer and 2μ for the resistance thermometer) were located at a distance of 200 μ, i.e., practically at the point, since, for example, $\rho_{u_1u'_1}=0.99$ for $\xi_1=\xi_2=$

0.2 mm. The simultaneous recording of the output signals of the thermometer and the hot-wire anemometer are necessary for the calculation of $\rho_{u_1tt}(\tau)$ by the method described above. The information from the thermometer and the hot-wire anemometer was fed into the computer through two channels and the quantities appearing in Eq. (13) were calculated. The results of the calculations were printed out. For example, Fig. 4 shows the correlation coefficient $\rho_{u_1tt'}(\xi_1)$ for $U_1 = 8$ m/sec. After determining $\rho_{u_1tt'}$, the third derivative with respect to ξ_1 was computed. Figure 5 shows the results of the calculation of S_{TDIF} with a mean-square error of 12%. It is clear that this coefficient is a rapidly varying function of the Péclet number. Keeping in mind the calculated values of $S_{TDIS}(P_L)$, the function (8) can be constructed. This function is shown in Fig. 5. The figure shows the asymptotic value obtained analytically for uniform isotropic fields. As is clear from the figure the function F_2 decreases with increasing P_L. One can assume that F_2 tends to approach an asymptotic value. However, in contrast with the corresponding function for the velocity field (cf. [3]) appreciably larger values of R_L than are reached in the experiment under consideration are required to verify the relation

$$\lim_{R_L \to \infty} F_2 \to \frac{16\sqrt{3}}{5} \tag{14}$$

with certainty. We note, however, that it is hardly possible to reach values of R_L greater than 10^4 under laboratory conditions of degenerate turbulence. Therefore, on the basis of the results presented above we assume that Eq. (14) is actually satisfied for a uniform temperature field.

NOTATION

$\overline{t^2}$, mean square of temperature fluctuations; ν, kinematic viscosity; $\varkappa$, thermal diffusivity; $\vec{\xi}$, separation vector of two points; $\Delta\xi$, Laplacian operator in $\vec{\xi}$ space; $\sigma = \nu/\varkappa$, Prandtl number; $\overline{q}^2 = \overline{u_i u_i}$, kinetic energy of turbulence; U, longitudinal flow velocity; $R_u = R_u(\vec{\xi}) = \overline{u(\mathbf{x}) \cdot u(\mathbf{x} + \vec{\xi})}$, spatial correlation of velocity fluctuations. Indices: *, isotropy; 0, $\vec{\xi} = 0$.

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